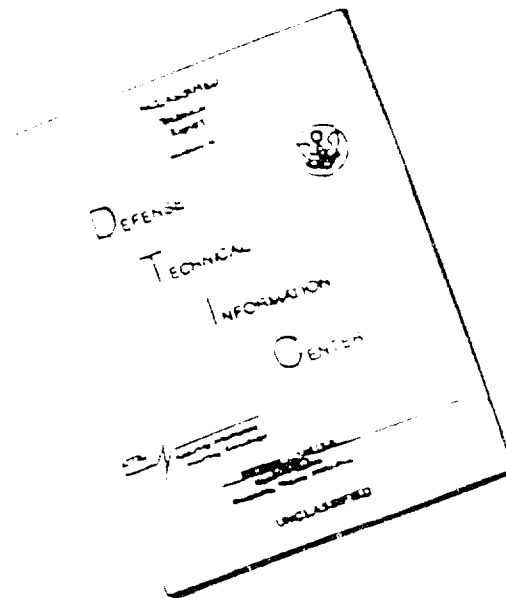# UNCLASSIFIED

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GENERALIZATION OF PARTIAL DOSAGE-AREA CURVES FROM AN INSTANTANEOUS POINT SOURCE AT GROUND LEVEL

O.R.G. Note Number 11

by   George H. Milly
Richard E. Heitman
Donald F. Molino

MAY 1962

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INTRODUCTION

1. Several methods for the generalization and simplification
of diffusion equations of the Sutton type have been advanced from
time to time in the form of nomographs, tables, and slide rules.
The equations which have been given major consideration are the
expressions for dosage versus downwind distance from the infinite
line source and the dosage-area distribution from the instantaneous
point source. The models for the normal line source, the uniform
finite line source, or any of the models which compute partial
dosage are not as easily represented by such techniques.

2. In this note, a method is derived for presenting the par-
tial dosage-area equation in the form of generalized curves. A
transposition of variables is introduced which permits the construc-
tion of generalized dosage-area distributions in the form of trans-
formed variables representing dosage and area. From equations in
terms of these transformed variables specific values of dosage and
area can be derived. All parameters with the exception of tempera-
ture gradient (stability), are reflected in the transformed varia-
bles. Consequently, a separate curve is plotted for each value of
this parameter.

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3. The basic equation being discussed is presented as equation (35) in ORG Study Nr. 17.[*] This equation is an approximation of the area covered by a dosage equal to or greater than D after the source cloud has traveled a time, t, at velocity $\bar{u}$. The derivation presented here is primarily algebraic. For the complete development of the starting equation, the definition of terms, and the underlying assumptions, reference should be made to the source document where these aspects are covered in greater detail.

4. The initial equation is given as follows:

$$A(D,t) = \frac{\sqrt{2\pi}\ \sigma_{y(x_1)}\ x_1}{(\alpha+1)\left(\frac{\alpha+1}{\alpha+\beta}\right)^{\frac{1}{2}}} \left[\frac{Q}{\pi\sigma_{y(x_1)}\ \sigma_{z(x_1)}\ \overline{u}D}\right]^{\frac{\alpha+1}{\alpha+\beta}}$$

$$\left\{1 - \operatorname{erf}\left[\ln\frac{\left(\frac{Q}{\pi\sigma_{y(x_1)}\ \sigma_{z(x_1)}\ \overline{u}D}\right)^{\frac{\alpha+1}{\alpha+\beta}}}{\left(\frac{\overline{u}t}{x_1}\right)^{\alpha+1}}\right]^{-\frac{1}{2}}\right.$$

$$\left. + \frac{2}{\sqrt{\pi}}\left[\ln\frac{\left(\frac{Q}{\pi\sigma_{y(x_1)}\ \sigma_{z(x_1)}\ \overline{u}D}\right)^{\frac{\alpha+1}{\alpha+\beta}}}{\left(\frac{\overline{u}t}{x_1}\right)^{\alpha+1}}\right]^{-\frac{1}{2}}\right\}$$

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[*]Milly, G.H. "Atmospheric Diffusion and Generalized Munitions Expenditures." ORG Study Nr. 17, U. S. Army Chemical Corps Operations Research Group, May 1958. Reissued in an unclassified version in March 1962.

$$\left[\frac{\left(\frac{\overline{u}t}{x_1}\right)^{\alpha+1}}{\left(\frac{Q}{\pi\sigma_{y(x_1)}\,\sigma_{z(x_1)}\,\overline{u}D}\right)^{\frac{\alpha+1}{\alpha+\beta}}}\right]\right\} \tag{1}$$

5. Consider the equation applied to a set of reference conditions denoted by subscript "0" for the following parameters:

$$Q_0 \quad = \quad 1 \text{ mg}$$

$$\sigma_{y_0}(x_1) \quad = \quad 1 \text{ meter}$$

$$\overline{u}_0 \quad = \quad 1 \text{ meter/min}$$

$$t_0 \quad = \quad 1 \text{ min}$$

6. Under other conditions, the parameter values are defined as:

$$Q \quad = \quad k_1\, Q_0$$

$$\sigma_{y(x_1)} \quad = \quad k_2\, \sigma_{y_0}(x_1)$$

$$\overline{u} \quad = \quad k_3\, \overline{u}_0$$

$$t \quad = \quad k_4\, t_0 \tag{2}$$

7. Then equation (1) can be rewritten as:

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$$A(D, k_4, t_0) = \frac{\sqrt{2}\pi\, k_2\, \sigma_{y_0}(x_1)\, x_1}{(\alpha+1)\left(\frac{\alpha+1}{\alpha+\beta}\right)^{\frac{1}{2}}} \left[\frac{k_1\, Q_0}{\pi\, k_2\, \sigma_{y_0}(x_1)\, \sigma_z(x_1)\, k_3\, \bar{u}_0 D}\right]^{\frac{\alpha+1}{\alpha+\beta}}$$

$$\left\{ 1 - \mathrm{erf}\left[\ln \frac{\left(\dfrac{k_1\, Q_0}{\pi k_2\, \sigma_{y_0}(x_1)\, \sigma_z(x_1)\, k_3\, \bar{u}_0 D}\right)^{\frac{\alpha+1}{\alpha+\beta}}}{\left(\dfrac{k_3\, \bar{u}_0\, k_4\, t_0}{x_1}\right)^{\alpha+1}}\right]^{\frac{1}{2}} \right.$$

$$+ \frac{2}{\sqrt{\pi}} \left[\ln \frac{\left(\dfrac{k_1\, Q_0}{\pi k_2\, \sigma_{y_0}(x_1)\, \sigma_z(x_1)\, k_3\, \bar{u}_0 D}\right)^{\frac{\alpha+1}{\alpha+\beta}}}{\left(\dfrac{k_3\, \bar{u}_0\, k_4\, t_0}{x_1}\right)^{\alpha+1}}\right]^{\frac{1}{2}}$$

$$\left. \left[\frac{\left(\dfrac{k_3\, \bar{u}_0\, k_4\, t_0}{x_1}\right)^{\alpha+1}}{\left(\dfrac{k_1\, Q_0}{\pi k_2\, \sigma_{y_0}(x_1)\, \sigma_z(x_1)\, k_3\, \bar{u}_0 D}\right)^{\frac{\alpha+1}{\alpha+\beta}}}\right] \right\}$$

(3)

8. The recurrent parenthetical term with exponent $(\alpha+1)/(\alpha+\beta)$ can be rewritten as follows:

$$\left[\frac{k_1 Q_0}{\pi k_2 \, \sigma_{y_0(x_1)} \, \sigma_{z(x_1)} \, k_3 \, \bar{u}_0 D} \right]^{\frac{\alpha+1}{\alpha+\beta}}$$

$$= \left[\left(\frac{Q_0}{\pi \sigma_{y_0(x_1)} \, \sigma_{z(x_1)} \, \bar{u}_0 \, \frac{k_2 k_3}{k_1} D} \right) \left(\frac{(k_3 k_4)^{\alpha+\beta}}{(k_3 k_4)^{\alpha+\beta}} \right) \right]^{\frac{\alpha+1}{\alpha+\beta}}$$

$$= (k_3 k_4)^{\alpha+1} \left[\frac{Q_0}{\pi \sigma_{y_0(x_1)} \, \sigma_{z(x_1)} \, \bar{u}_0 \, \frac{k_2 k_3}{k_1} (k_3 k_4)^{\alpha+\beta} D} \right]^{\frac{\alpha+1}{\alpha+\beta}}$$

$$(4)$$

9. The two logarithmic terms in equation (3) are identical and can be rewritten as follows:

$$\frac{\left(\dfrac{Q_0}{\pi \sigma_{y_0(x_1)} \, \sigma_{z(x_1)} \, \bar{u}_0 \, \frac{k_2 k_3}{k_1} D} \right)^{\frac{\alpha+1}{\alpha+\beta}}}{\left(\dfrac{\bar{u}_0 t_0}{x_1} \right)^{\alpha+1} \left(k_3 k_4 \right)^{\alpha+1}}$$

$$= \frac{\left(\dfrac{Q_0}{\pi \sigma_{y_0(x_1)} \, \sigma_{z(x_1)} \, \bar{u}_0 \, \frac{k_2 k_3}{k_1} (k_3 k_4)^{\alpha+\beta} D} \right)^{\frac{\alpha+1}{\alpha+\beta}}}{\left(\dfrac{\bar{u}_0 t_0}{x_1} \right)^{\alpha+1}}$$

$$(5)$$

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10. The last bracketed term in equation (3) is just the reciprocal of the logarithmic terms.

11. We define a pseudo-dosage as:

$$D_r = \frac{k_2 k_3}{k_1} (k_3 k_4)^{\alpha+\beta} D$$

$$(6)$$

and a pseudo-area as:

$$A_r = \frac{A}{k_2 (k_3 k_4)^{\alpha+1}}$$

$$(7)$$

By reference to (4) and (5), equation (3) can be rewritten as:

$$A_r (D_r, t_0) = \frac{\sqrt{2\pi}\, \sigma_{y_0}(x_1)\, x_1}{(\alpha+1)\left(\frac{\alpha+1}{\alpha+\beta}\right)^{\frac{1}{2}}} \left[\frac{Q_0}{\pi \sigma_{y_0}(x_1)\, \sigma_z(x_1)\, \bar{u}\, D_r} \right]^{\frac{\alpha+1}{\alpha+\beta}}$$

$$\left\{ 1 - \mathrm{erf} \left[\frac{\ln\left(\frac{Q_0}{\pi \sigma_{y_0}(x_1)\, \sigma_z(x_1)\, \bar{u}_0\, D_r} \right)^{\frac{\alpha+1}{\alpha+\beta}}}{\left(\frac{\bar{u}_0 t_0}{x_1} \right)^{\alpha+1}} \right]^{\frac{1}{2}} \right.$$

$$\left. + \frac{2}{\sqrt{\pi}} \left[\frac{\ln\left(\frac{Q_0}{\pi \sigma_{y_0}(x_1)\, \sigma_z(x_1)\, \bar{u}_0\, D_r} \right)^{\frac{\alpha+1}{\alpha+\beta}}}{\left(\frac{\bar{u}_0 t_0}{x_1} \right)^{\alpha+1}} \right]^{\frac{1}{2}} \right.$$

$$\left\{ \left[\frac{\left(\dfrac{\bar{u}_0 t_0}{x_1} \right)^{\alpha+1}}{\left(\dfrac{Q_0}{\pi \sigma_{y_0}(x_1)\, \sigma_z(x_1)\, \bar{u}_0\, D_r} \right)^{\frac{\alpha+1}{\alpha+\beta}}} \right] \right\}$$

<div align="right">(8)</div>

12. If the coefficients defined by equation (2) are substituted in the expressions for the pseudo-variables, it can be seen that equation (8) is identical to equation (1).

13. Since the reference values of the parameters have been taken as unity upon substitution for the coefficients in (6) and (7) according to (2) we obtain

$$D_r = \frac{\sigma_{y(x_1)}\, (\bar{u}t)^{\alpha+\beta}\, \bar{u}\, D}{Q}$$

<div align="right">(9)</div>

and

$$A = A_r\, \sigma_{y(x_1)}\, (\bar{u}t)^{\alpha+1}$$

<div align="right">(10)</div>

14. By employing the reference parameter values, a plot of D_r vs A_r can be constructed which can be used with equations (9) and (10) to provide a general solution for any value of D and A.

APPLICATION

15. Since the primary use of equation (8) is the estimation of partial dosage-area for individual bursting munitions, the values of α have been taken from the curves in ORG Study Nr. 17

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which were derived directly from tests of these items. These values of α are given the notation α' in the original report and must be employed with the values of $\sigma_{y(x_1)}$ which are designated as $\sigma'_{y(x_1)}$, and which are a function of the munition filling weight. The use of these values in no way limits the preceding development of the general equation, but the particular values are α' which have been used to construct the curves in Figure 1 restrict their use accordingly.

16. Figure 1 is a plot of D_r and A_r for unit values of the reference parameters and a range of temperature gradient conditions. The diffusion parameters employed are as follows:

ΔT	α'	β	$\sigma_{z(x_1)}$
+2	.20	.66	3.85
+1	.25	.74	4.35
0	.30	.88	5.85
-1	.40	1.18	11.00
-2	.60	1.66	25.00

17. Figure 1 can be used by calculating D_r for the conditions of interest from equation (9); a value of A_r is then obtained from the graph for the appropriate temperature gradient, and the area, A, is calculated from A_r using equation (10). For convenience in calculating D_r and A, Figures 2 and 3 have been prepared. These are plots of $(\bar{u}t)$ vs $(\bar{u}t)^{\alpha'+\beta}$ and $(\bar{u}t)$ vs $(\bar{u}t)^{\alpha'+1}$, respectively, for each of the five values of temperature gradient given in Figure 1.

18. It should be remembered that, as in ORG Study Nr. 17, the above equations are valid only when the condition

$$\left(\frac{\bar{u}t}{x_1}\right)^{\alpha+1} \leq \left(\frac{Q}{\pi\sigma_{y(x_1)}\,\sigma_{z(x_1)}\,\bar{u}D}\right)^{\frac{\alpha+1}{\alpha+\beta}}$$

is met, otherwise the total dosage equation must be used. In Figure 1, the total dosage relation is represented by the straight line segment of the curves.

NUMERICAL EXAMPLE

19. Find the area covered to a dosage of 100 mg min/m^3, given the following parameter values:

$$\text{wind speed, } \bar{u} = 107.27 \text{ m/min}$$
$$\text{time, } t = \tfrac{1}{2} \text{ min}$$
$$\text{airborne agent, } Q = 1.635 \times 10^6 \text{ mg}$$
$$\text{width parameter, } \sigma'_{y(x_1)} = 14.4 \text{ m}$$
$$\text{neutral temperature gradient, } \Delta T = 0°F$$

20. From equation (9), $D_r = \dfrac{\sigma'_{y(x_1)}\,(\bar{u}t)^{\alpha'+\beta}\,\bar{u}D}{Q}$

Since $(ut) = 53.6$, from Figure 2, $(\bar{u}t)^{\alpha'+\beta} = 113$

$$D_r = \frac{14.4\,(113) \times 107.27 \times 100}{1.635 \times 10^6} = \frac{1.745 \times 10^7}{1.635 \times 10^6} = 10.67$$

From Figure 1, at $D_r = 10.67$

$$A_r = 0.54$$

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From equation (10):

$$A = A_r \, \sigma'_{y(x_1)} \, (\bar{U}t)^{\alpha'+1}$$

and from Figure 3, $(\bar{U}t)^{\alpha'+1} = 170$

$$A = 0.54 \times 14.4 \times 170$$

$$A = 1,322 \text{ sq m}$$

Fig. 1. - Scaled Dosage (D_r) vs Scaled Area (A_r)

Fig. 2. - $(ut)^{\alpha'+\beta}$ vs (ut) for Various Temperature Gradients, $\Delta T = T_{4m} - T_{0.5m}$ in $^\circ F$

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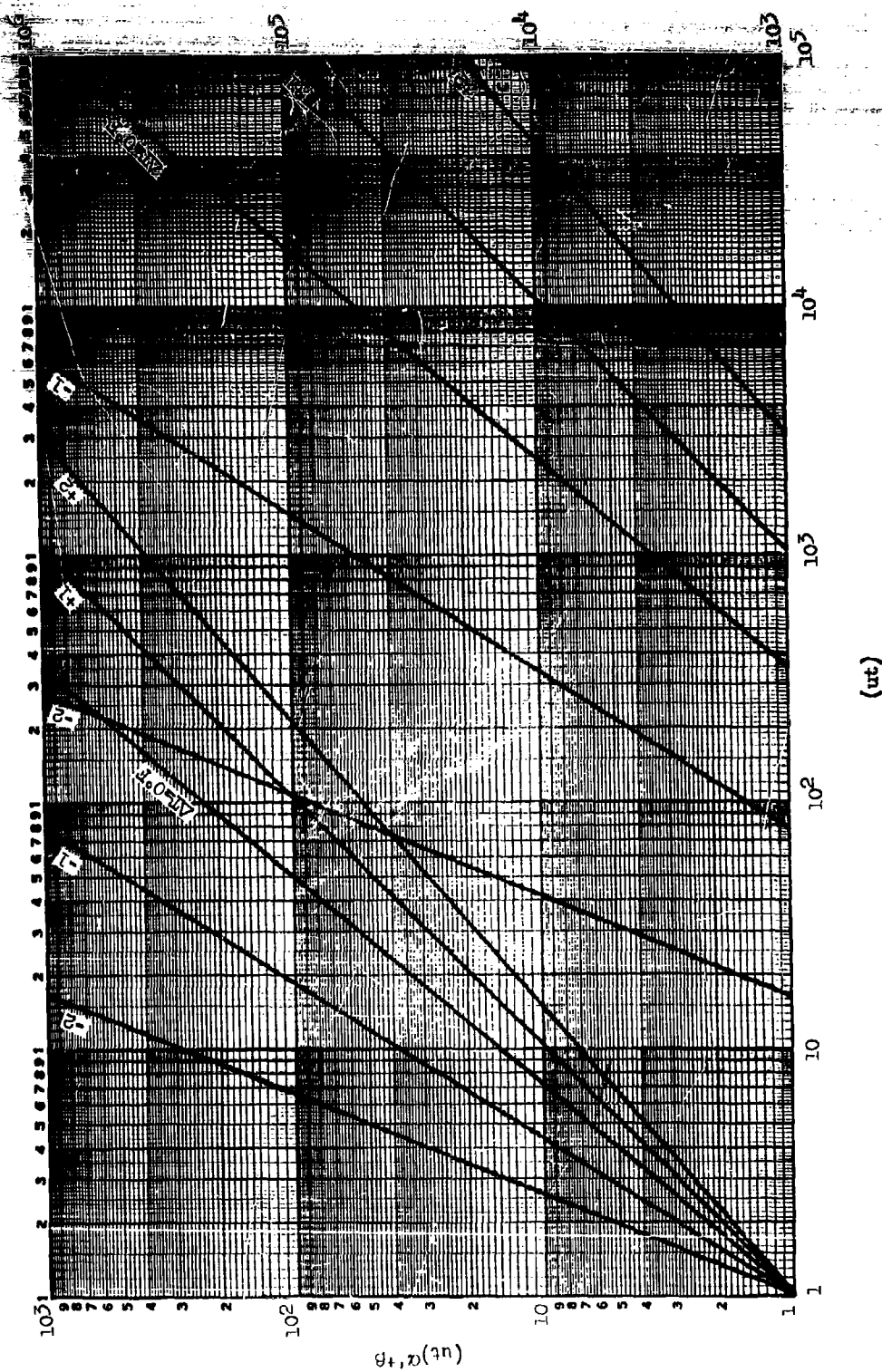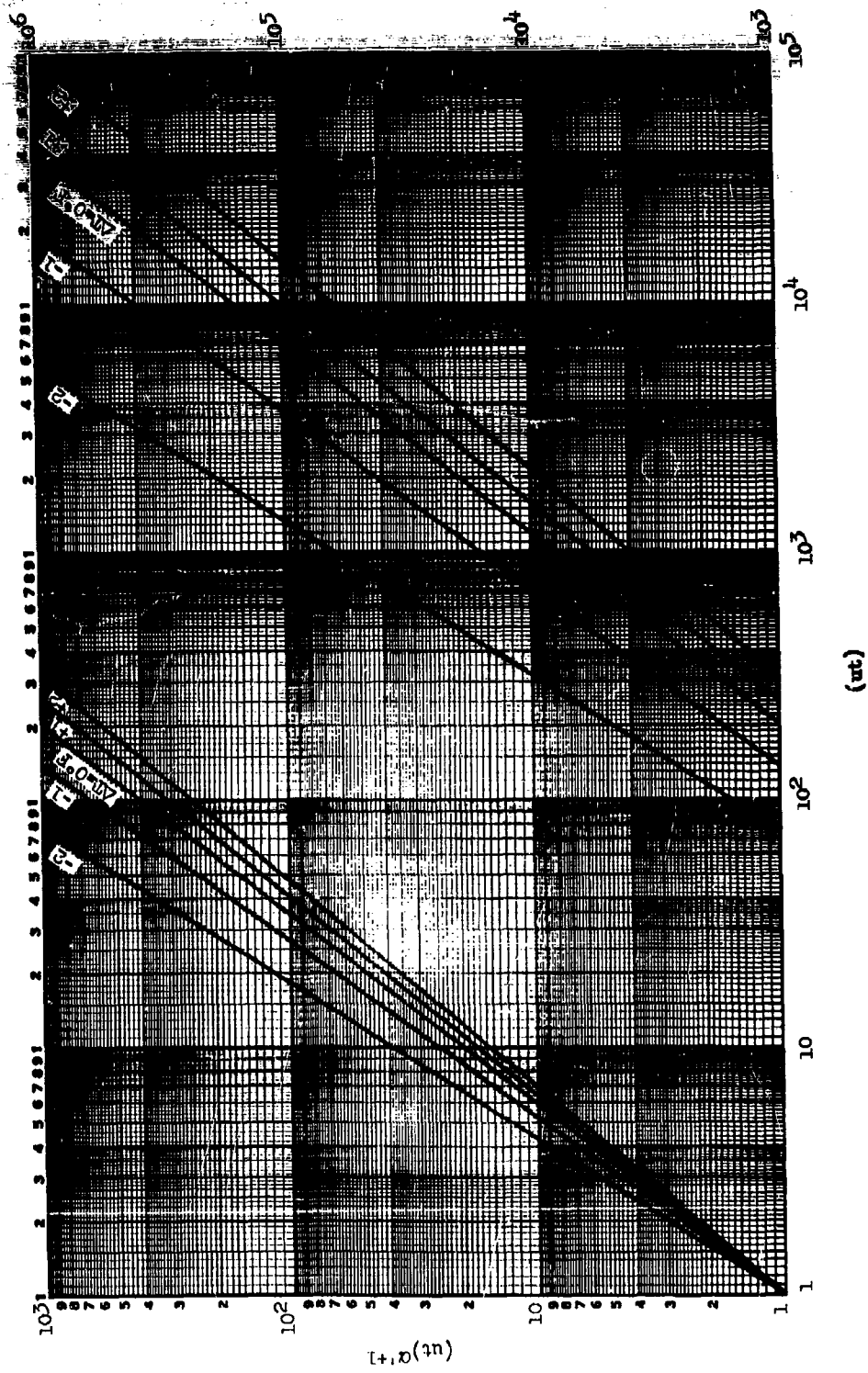
Fig. 3. - $-(ut)^{\alpha'+1}$ vs (ut) for Various Temperature Gradients, $\Delta T = T_{hm} - T_{0.5m}$ in $^\circ F$

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"Generalization of Point Source at Cml Corps Operations

Geo. H. Milly, R. E. Heitman, & D. F. Molino. "Generalization of Partial Dosage-area Curves from an Instantaneous Point Source at Ground Level," ORG Note No. 11. U. S. Army Cml Corps Operations Research Group, Army Cml Center, Md. May 1962.

Micrometeorology
Cloud Diffusion
Cloud Travel
Instantaneous Point Source
Ground Level Source
Partial Dosage-area

A method is derived for presenting the partial dosage-area equation in the form of generalized curves. A transposition of variables is introduced which permits construction of generalized dosage-area distributions. From equations in terms of these transformed variables specific values of dosage and area can be derived. All parameters except temperature gradient (stability) are reflected in the transformed variables. Curves of the transformed variables are given for various temperature gradients.

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